

# Using Rough Sets Techniques as a Fault Diagnosis Classifier for Induction Motors

E. L. Bonaldi, L. E. Borges da Silva\*, G. Lambert-Torres,  
L. E. L. Oliveira and F. O. Assunção

\* Electronic Engineering Department  
Escola Federal de Engenharia de Itajubá  
37500-090 – Itajubá – MG – Brazil  
[leborges@ice.efei.br](mailto:leborges@ice.efei.br)

**Abstract** — This paper describes the ongoing research on Rough Sets based classifier applied to Induction Motors fault diagnosis through Motor Current Signature Analysis (MCSA). The results of mechanical failures detection and how a Rough Sets based classifier is used as a monitoring system using current signature analysis in predictive maintenance are described in this paper.

## I. INTRODUCTION

Nowadays there is a great concern about the reliability of the productive process in order to reduce production costs and increase productivity in the industrial area. This fact makes maintenance techniques a very important issue. The highlights of the moment are predictive maintenance techniques. These techniques consist of using continuous monitoring systems. Their use is justified in the presence of failures, in general randomly, of complex equipment with a large amount of components. Such failures usually have serious economic or human consequences.

Since induction motors are often critical components in the industrial process, they deserve special attention from the plant maintenance department. This paper presents a method of improving fault detection in induction motors through current signature analysis using a Rough Sets based Classifier.

Normally, the number of information available to reach the proper diagnosis is large enough to complicate fast human analysis. This is not an easy task for the expert. It is even more complicate for a technician to deal with all available data, normally a huge number of measurements that must be manipulated and clustered in order to visualize the current state of the equipment. In this particular point, the use of Rough Sets helps the human operator to cope with all available information and cluster it in a reasonable and comprehensible way, which is normally done in ordinary Expert Systems.

This paper presents an ongoing approach to fault detection and diagnosis that copes with the analysis performed by the classifier and tries to make a classification with two outputs. The first output is the failure mode and the second one is the operational mode in one of the three states, namely: normal, warning, and emergency. In the first state, all signals and all measurements are within the nominal rates. In the second state, all signals continue to be acceptable although some of the measurements may be above the nominal rates. For the

emergency operational state, the signals are above the nominal rates and the maintenance is mandatory. The primary results obtained by the application of the methodology of Rough Sets give us the hope that this technique can be used as a powerful tool toward a robust classifier in fault diagnosis.

This paper is divided in three parts. The first one gives a concise description of the Motor Current Signature Analysis approach and the mechanical faults that have been used. The second part shows an example of laboratory tests. The third part presents the application of Rough Sets based classifier and the results obtained.

## II. OVERVIEW OF MOTOR CURRENT SIGNATURE ANALYSIS AND THE DETECTED FAULTS

MCSA is a noninvasive technique which diagnosis problems in induction motors. It consists of utilizing the results of spectral analysis of the stator one-phase current signal. When a failure is present, the frequency spectrum of the line current becomes different from that of a non-faulted one. Such fault modulates the air-gap and produces rotating frequency harmonics in the self and mutual inductances of the machine. Since the flux linkages oscillate at only the electric supply frequency, these harmonic inductances result in stator current harmonic at rotating frequency sidebands of the line frequency [1].

The characteristic frequencies of failure are very well known and have been described by many authors. However, this paper section intends to introduce in a concise manner the general idea of the theory, and because of that, the characteristic frequencies of each studied mechanical fault will be presented.

### A. Rotor Asymmetry

When a rotor asymmetry is present in an induction motor, the air-gap flux density is disturbed. This disturbance rotates at shaft speed, generating characteristic components in the frequency spectrum given by [2]:

$$f_{as} = f \left[ k \left( \frac{1-s}{p/2} \right) \pm s \right] \quad (1)$$

Where:  $f$  is the supply frequency;  $s$  is the slip per unit;  $p$  is the number of poles; and  $k = 1, 2, 3, \dots$

### B. Rotor unbalance

In the case of dynamic eccentricity that varies with the rotor position, what happens is an oscillation in the air-gap length, causing variations in the air-gap flux. This fact, in turn, affects the machine instantaneous inductance, producing stator current harmonics in [2]:

$$f_{ru} = f \left[ k \left( \frac{1-s}{p/2} \right) \pm 1 \right] \quad (2)$$

### C. Air-gap eccentricity

There are two methods for the detection of air-gap eccentricity. The first one monitors the behavior of the sidebands present in the current spectrum near the slot frequency. The associate frequencies with this failure using this first method, are given by:

$$f_{slot/ecc} = f \left[ (kR \pm n_{ecc}) \cdot \left( \frac{1-s}{p/2} \right) \pm n_s \right] \quad (3)$$

Where:  $R$  is the number of rotor bars;  $n_{ecc}$  is the eccentricity order number;  $n_s$  is the supply frequency harmonic rank; and  $k = 1, 2, 3, \dots$

The great disadvantage of this method is the need for the constructive aspects of the machine. On the other hand, by using this method of monitoring, it is possible to separate broken bar effects from eccentricity effects.

The second method consists of monitoring the behavior of the sidebands around the fundamental frequency. These failure characteristic frequencies are given by:

$$f_{ecc} = f \left[ 1 \pm k \left( \frac{1-s}{p/2} \right) \right] \quad (4)$$

The great advantage of this second approach is that it is not necessary to know the rotor construction aspects to accomplish an evaluation of the health of the motor [3].

### D. Broken Bars

The detection of broken bars through the stator current spectrum can be accomplished by observing two particular components around the fundamental component.

When broken bars are present, the current spectrum presents two components equally spaced of  $2 \cdot f_s$  from the fundamental frequency. The left component ( $f - 2f_s$ ) results from the failure. The right component results from the speed ripple [4].

This way, the characteristic frequencies of broken bars are given by [5]:

$$f_{bq} = f(1 \pm 2fs) \quad (5)$$

### E. Bearing damages

The monitoring of bearing damages is very important in a predictive maintenance system since they are responsible for 40% of the failures in induction machines [6].

There are several causes for bearing damages. Since this is not the objective of this work, the paper will present just the characteristic components of failure in the outer and inner races, and rolling elements. The formulations of these characteristic components depend on the bearing dimensions. Fig. 1 presents the dimensions involved in the frequency calculations:

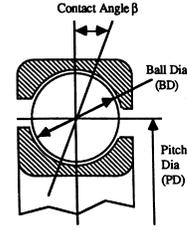


Fig 1: Bearing dimensions

The characteristic frequencies of failures in the rolling element, inner race and outer race are respectively given by [7, 8]:

$$f_{re} = \left| f \pm m \cdot \frac{PD}{BD} \cdot f_{rm} \left[ 1 - \left( \frac{BD}{PD} \cdot \cos \beta \right)^2 \right] \right| \quad (6)$$

$$f_{ir} = \left| f \pm m \cdot \frac{n}{2} \cdot f_{rm} \left( 1 + \frac{BD}{PD} \cdot \cos \beta \right) \right| \quad (7)$$

$$f_{or} = \left| f \pm m \cdot \frac{n}{2} \cdot f_{rm} \left( 1 - \frac{BD}{PD} \cdot \cos \beta \right) \right| \quad (8)$$

However, the characteristic race frequencies can be approximated for most bearings with between six and twelve balls by:

$$f_{ir} = f \pm 0.6 \cdot n \cdot f_{rm} \quad (9)$$

$$f_{or} = f \pm 0.4 \cdot n \cdot f_{rm} \quad (10)$$

Where:  $f_{rm}$  is the rotor speed in hertz;  $n$  is the number of rolling elements; and  $m = 1, 2, 3 \dots$

## III. Laboratory Tests

This work investigates some mechanical faults such as air-gap eccentricity, load unbalances, broken bars and bearing damage. Laboratory tests have confirmed the technique efficiency in monitoring the status of three-phase-induction motors. One of the tests conducted are described below in order to provide a better understanding of the approach.

### A. Load Unbalance

A load unbalance was imposed to the motor by installing a metal disc with three holes in the shaft. Three levels of failure severity were created by adding a small mass of 80 grams in each hole placed 70 mm ( $d_1$ ), 90 mm ( $d_2$ ) and 110 mm ( $d_3$ ) from the shaft respectively. Fig. 2 illustrates the process.

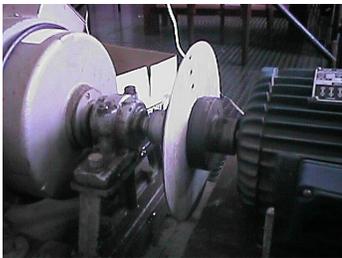


Fig. 2: Laboratory assembly to simulate load unbalance

The frequency spectra were obtained and the tendency curve was plotted. Fig. 3a presents the frequency spectrum of normal condition and Fig. 3b presents the frequency spectrum of level-three severity. Fig. 4 presents the tendency curve.

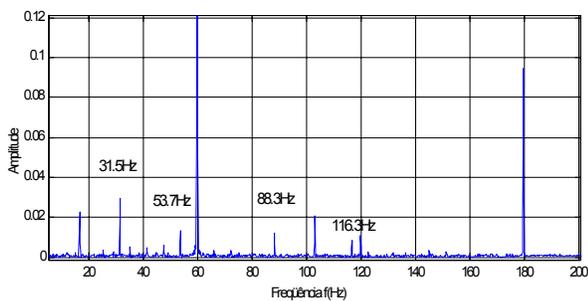


Fig. 3a: Spectrum of normal condition

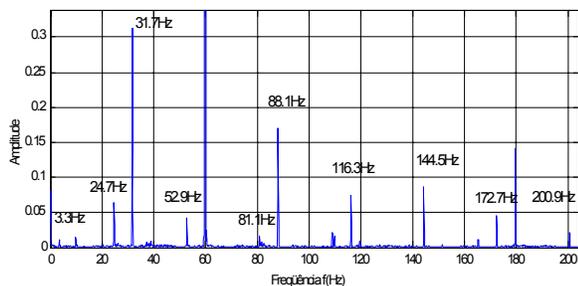


Fig 3b: Spectrum of level-three severity

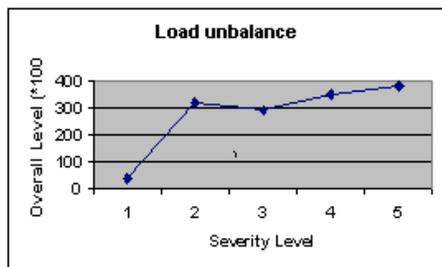


Fig. 4: Tendency curve

The severity levels are: 1) non fault condition; 2) motor plus unbalanced disc without mass; 3) mass placed in  $d_1$ ; mass placed in  $d_2$ ; mass placed in  $d_3$ .

## B. Bearing damage

In order to analyze the effects of bearing damage, a hole was drilled through the outer race as Fig. 5 illustrates. According to the equation (10), presented in section II, the first three pairs of characteristic frequencies were 43.3 and 162.7 Hz, 146.3 (component that responded to the failure) and 265.7 Hz, 249.3 and 368.7 Hz. Fig 6 presents the obtained spectra with zoom in the area where a characteristic component appeared.



Fig. 5: Bearing with a hole in the outer race

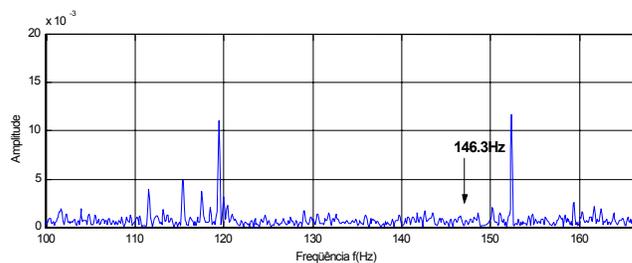


Fig. 6a: Spectrum of normal condition

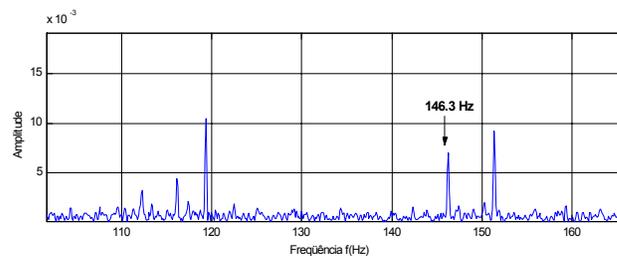


Fig. 6b: Stator current spectrum of a motor with a hole in the outer race of the shaft-end bearing

The figures show that the MCSA approach responded very well to the failure severity. Other tests also had the same good response and this push the research toward a development of a robust and reliable classifier to fault diagnosis.

## IV. Rough Sets Based Classifier

### A. Overview of Rough Set Theory

The objective of this paper section is to present the fundamental concepts of the rough set theory. The main idea

is to transform a set of examples in a set of rules that represents the operation state of an induction motor.

## 1. Information System

An information system can be defined as a 4-tuple  $K=(U, \mathbf{R}, V, \rho)$ , where  $U$  is a finite set of objects (search space),  $\mathbf{R}$  is a finite set of attributes (state of each signal, currents and vibration),  $V$  is the domain of each attribute of  $\mathbf{R}$ , and  $\rho$  is a total function (named information function) that defines the following application:  $\rho: U \times \mathbf{R} \rightarrow V$ , i.e., the examples.

The concept of information system is not exclusive of the rough set theory and has been extensively used in information theory.

## 2. Approximation Sets

One of the main contributions of Rough Set Theory is to automatically transform data into knowledge [9]. This theory uses lower and upper approximation of a set, as shown in Fig. 7 [10]. According to this figure we can observe five regions (or sets) of interest:  $\bar{R}X$  and  $\underline{R}X$ , and  $POS_R(X)$ ,  $BN_R(X)$  and  $NEG_R(X)$ . Each one of these is defined below.

Let a set  $X \subseteq U$ ,  $R$  be an equivalence relation, and  $K=(U, \{R\})$  be a knowledge base. Two subsets can be associated to these:

- a) R-lower:  $\underline{R}X = \bigcup \{Y \in U/R : Y \subseteq X\}$
- b) R-upper:  $\bar{R}X = \bigcup \{Y \in U/R : Y \cap X \neq \emptyset\}$

These definitions mean that the elements that belong to the  $\underline{R}X$  set can be, with certainty, classified as elements of  $X$ ; while the elements belonging to the  $\bar{R}X$  set can be, only possibly, classified as elements of  $X$ .

In the same way,  $POS_R(X)$ ,  $BN_R(X)$  and  $NEG_R(X)$  are defined as [11]:

- c)  $POS_R(X) = \underline{R}X \Rightarrow$  certainly member of  $X$
- d)  $NEG_R(X) = U - \bar{R}X \Rightarrow$  certainly non-member of  $X$
- e)  $BN_R(X) = \bar{R}X - \underline{R}X \Rightarrow$  possibly member of  $X$

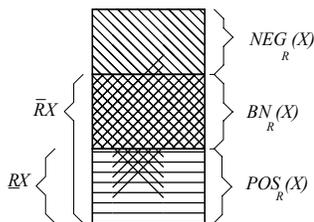


Fig. 7: Definition of R-approximation sets and R-regions.

Based on the above definitions, the concept of accuracy measure ( $\alpha_R(X)$ ) can now be presented, which numerically characterizes the inaccuracy of the knowledge, using the cardinality of  $\bar{R}X$  and  $\underline{R}X$  sets, i.e.,

$$\alpha_R(X) = \frac{\text{card } \underline{R}X}{\text{card } \bar{R}X} \quad (11)$$

where  $\alpha_R(X)$  is defined in the interval  $[0,1]$ .

When  $\alpha_R(X)=1$ , the set  $X$  is named R-definable, and the  $BN_R(X)$  region is empty. In this case, the rough set theory is reduced to classical Cantor set theory.

## 3. Reduct and Core of Knowledge

The concepts of reduct and core are important in the knowledge base reduction. Let  $\mathbf{R}$  be a family of equivalence relations. The reduct of  $\mathbf{R}$ ,  $RED(\mathbf{R})$ , is defined as a reduced set of relations that conserves the same inductive classification of set  $\mathbf{R}$ . The core of  $\mathbf{R}$ ,  $CORE(\mathbf{R})$ , is the set of relations that appears in all reduct of  $\mathbf{R}$ , i.e., the set of all indispensable relations to characterize the relation  $\mathbf{R}$ .

### B. Knowledge Base Reduction

One of the most common approaches to get knowledge from an expert is by examples. The idea behind the knowledge base reduction is a simplification of this set of examples. This can be obtained with the following procedure:

- a) Calculate the core of the problem
- b) Eliminate (or substitute) a variable using another one; and
- c) Redefine the problem using new basic categories.

The algorithm that provides the reduction of conditions has been proposed in [11,12] and can be represented by the following steps:

- Step 1: Eliminate the dispensable attributes.
- Step 2: Compute the core of each example.
- Step 3: Compose a table with reduct value.
- Step 4: Merge possible examples.

### C. Algorithm Extension

A more sophisticated system for fault diagnosis can be developed redefining the band for the attributes related to each acquired values of analog variables according to a certain metric, creating a new and more flexible database. By applying the algorithm described before in this modified database, it is possible to obtain a more detailed and specific classification of the operating point of the Induction Motor under study.

Before the presentation of the algorithm, we need to remember two major concepts in Rough Set Theory: *reduct* and *core*. These concepts are important in the knowledge base reduction.

Let  $\mathbf{R}$  be a family of equivalence relations. The *reduct* of  $\mathbf{R}$ ,  $RED(\mathbf{R})$ , is defined as a reduced set of relations that conserves the same inductive classification of set  $\mathbf{R}$ . The

core of  $\mathbf{R}$ ,  $CORE(\mathbf{R})$ , is the set of relations that appears in all reduct of  $\mathbf{R}$ , i.e., the set of all indispensable relations to characterize the relation  $\mathbf{R}$ .

The algorithm that provides the reduction of conditions has been proposed in [11, 12], and can be represented by the following steps:

*Previous Steps:*

- Transform continuous values in ranges.
- Eliminate identical attributes.
- Eliminate identical examples.

*Step 1:* Eliminate dispensable attributes.

*Step 2:* Compute the core of the decision table.

*Step 3:* Compose a table with reduct value.

*Step 4:* Merge possible examples.

*Final Step:* Compose the final set of rules.

#### D. Description of the problem

The idea is to transform a set of examples in a set of rules that specifies the kind of failure (Broken bars, bearing damage, air-gap eccentricity) and represents the operational state of an induction motor. For the sake of explanation, some assumptions and reductions are made.

The operational state of the induction motor and the failure mode, shown in Fig. 8, depend on the information obtained from the data acquired from one phase of the stator current (shaft speed, characteristic frequencies, etc), parameters related to motor features (rated power, current, voltage and speed) and other attributes generated by other digital signal processing techniques. These attributes are used in a decision table in order to provide a more suitable way of accomplishing the Rough Sets Algorithm.

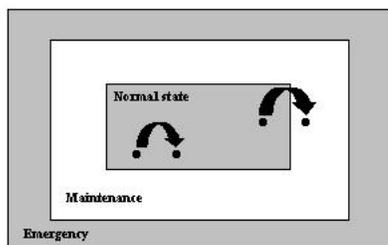


Fig. 8: Operational state of an Induction motor and changing of operational point

Table 1 presents a partial set of examples relating the different attributes with their values categorized in ranges.

Table 1: Set of examples

	A	B	C	D	E	F	G	H	O1	O2
1	N	N	N	S	UR	UL	C1	N	N	N
2	N	N	N	S	UR	UL	C1	C	N	N
3	N	N	N	S	UR	UL	C1	G	N	N
4	N	N	N	S	UR	UL	C1	N	N	N
5	N	N	N	S	UR	UL	C1	C	N	N
6	N	N	N	S	UR	UL	C1	G	N	N
7	N	N	N	S	UR	UL	C2	N	N	N
8	N	N	N	S	UR	UL	C2	C	N	N

9	N	N	N	S	UR	UL	C2	G	N	N
10	N	N	N	S	UR	UL	C2	N	N	N
11	N	N	N	S	UR	UL	C2	C	N	N
12	N	N	N	S	UR	UL	C2	G	N	N
13	N	N	N	R	R	R	C1	N	N	N
14	N	N	N	R	R	R	C1	C	N	N
15	N	N	N	R	R	R	C1	G	N	N
16	N	N	N	R	R	R	C1	N	N	N
17	N	N	N	R	R	R	C1	C	N	N
18	N	N	N	R	R	R	C1	G	N	N
19	N	N	N	R	R	R	C2	N	N	N
20	N	N	N	R	R	R	C2	C	N	N
21	N	N	N	R	R	R	C2	G	N	N
22	N	N	N	R	R	R	C2	N	N	N
23	N	N	N	R	R	R	C2	C	N	N
24	N	N	N	R	R	R	C2	G	N	N
25	W	N	N	S	UR	UL	C1	N	Ecc	W
26	W	N	N	S	UR	UL	C2	N	Ecc	W
27	W	N	N	R	R	R	C1	N	Ecc	W
28	W	N	N	R	R	R	C2	N	Ecc	W
29	W	N	N	F	OR	OL	C1	N	Ecc	W
30	W	N	N	F	OR	OL	C2	N	Ecc	W
31	E	N	W	S	UR	UL	C1	N	Ecc	E
32	E	N	W	S	UR	UL	C2	N	Ecc	E
33	E	N	W	R	R	R	C1	N	Ecc	E
34	E	N	W	R	R	R	C2	N	Ecc	E
35	E	N	W	F	OR	OL	C1	N	Ecc	E
36	E	N	W	F	OR	OL	C2	N	Ecc	E
37	N	W	N	S	UR	UL	C1	N	B	W
38	N	W	N	S	UR	UL	C2	N	B	W
39	N	W	N	R	R	R	C1	N	B	W
40	N	W	N	R	R	R	C2	N	B	W
41	N	W	N	F	OR	OL	C1	N	B	W
42	N	W	N	F	OR	OL	C2	N	B	W
43	W	E	N	S	UR	UL	C1	N	B	E
44	W	E	N	S	UR	UL	C2	N	B	E
45	W	E	N	R	R	R	C1	N	B	E
46	W	E	N	R	R	R	C2	N	B	E
47	W	E	N	F	OR	OL	C1	N	B	E
48	W	E	N	F	OR	OL	C2	N	B	E
49	N	N	W	S	UR	UL	C1	N	BB	W
50	N	N	W	S	UR	UL	C2	N	N	N
51	N	N	W	R	R	R	C1	N	BB	W
52	N	N	W	R	R	R	C2	N	N	N
53	N	N	W	F	OR	OL	C1	N	BB	W
54	N	N	W	F	OR	OL	C2	N	N	N
55	W	N	E	S	UR	UL	C1	N	BB	E
56	W	N	E	S	UR	UL	C2	N	BB	E
57	W	N	E	R	R	R	C1	N	BB	E
58	W	N	E	R	R	R	C2	N	BB	E
59	W	N	E	F	OR	OL	C1	N	BB	E
60	W	N	E	F	OR	OL	C2	N	BB	E

The condition attributes are:

- Specific current components:
    - A – Overall level of those components related to eccentricity;
    - B – Overall level of those components related to bearing damage;
    - C – Overall level of those components related to broken bars.
- A, B and C are classified in one of the three ranges, namely: Normal (N), Warning (W) or Emergency (E).

- Motor features:

D – slip;

E – fundamental current amplitude;

F – input power;

D is classified as fast (F), rated (R) or slow (S). E is classified as underrated (UR), rated (R) or Overrated (O). F, in turns, is classified as underload (UL), rated (R) or Overload (OL). D, E and F are per unit values related to rated values.

- General Attributes:

G – related to kind of load;

H - related to load and environment conditions.

G is classified as C1 (constant load) or C2 (pulsating load). H is classified as good (G), normal (N) or critical (C).

The decisions attributes are:

O1 - Failure mode (air-gap eccentricity (Ecc), bearing damage (B) or broken bars (BB));

O2 - Failure severity (Normal (N), Warning (W) or Emergency (E)).

The set of examples of table 1, when finally reduced, generates the following reduced set presented in table 2:

Table 2: Reduced Set

	A	B	C	G	O1	O2
1	N	N	N	-	N	N
2	N	N	-	C2	N	N
3	W	N	N	-	Ecc	W
4	E	-	-	-	Ecc	E
5	-	W	-	-	B	W
6	-	E	-	-	B	E
7	N	-	W	C1	BB	W
8	N	-	W	C2	N	N
9	-	-	E	-	BB	E

The indication “-“, in some cells, means that the attribute is unnecessary for the classification.

Final Step: according to the table 2, one can express the knowledge present in table 1 by the following set of rules:

**If (A is N and B is N and C is N) or (A is N and B is N and G is C2) or (A is N and C is W and G is C2) then (O1 is N and O2 is N)**

**If (A is W and B is N and C is N) then (O1 is Ecc and O2 is W)**

**If (A is E) then (O1 is Ecc and O2 is E)**

**If (B is W) then (O1 is B and O2 is W)**

**If (B is E) then (O1 is B and O2 is E)**

**If (A is N and C is W and G is C1) then (O1 is BB and O2 is W)**

**If (C is E) then (O1 is BB and O2 is E)**

## V. CONCLUSION

This paper presents the results of a systematic approach to detect superfluous input variables and unnecessary

conditions in a set of examples in order to classify specific problems of induction motors. The size of the knowledge base represents an important challenge for reliable diagnosis in Predictive Maintenance. In this case, a systematic and rational reduction of a knowledge base, by keeping only the core of the knowledge is desirable.

The method described in this paper to reduce the knowledge base is based on Rough Set theory, and it tries to create an automatic approach to transform data into knowledge. The methodology developed is applied to induction motor predictive maintenance. Although the described technique is still under development, the obtained results are encouraging.

This way, in a first evaluation of the technique applied to predictive maintenance in induction motor, the researchers can say that rough sets theory is a very promising tool to generate the rules which will diagnose the status of the motor.

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